Quantum localization in circularly polarized electromagnetic field in ultra-strong field limit

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Abstract: We predict analytically and confirm numerically the existence of sharply localized quantum states in an ultra-strong circularly polarized electromagnetic field with the probability density that represents non-classical wave packets moving around strongly unstable classical circular orbits.

OCIS codes: (020.0020) Atomic and molecular physics; (270.6620) Strong-field processes

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A Hydrogen atom in a Circularly Polarized (CP) electromagnetic field has been a subject of growing interest [1, 2, 3]. The discovery of so-called Trojan wavepackes [4] has

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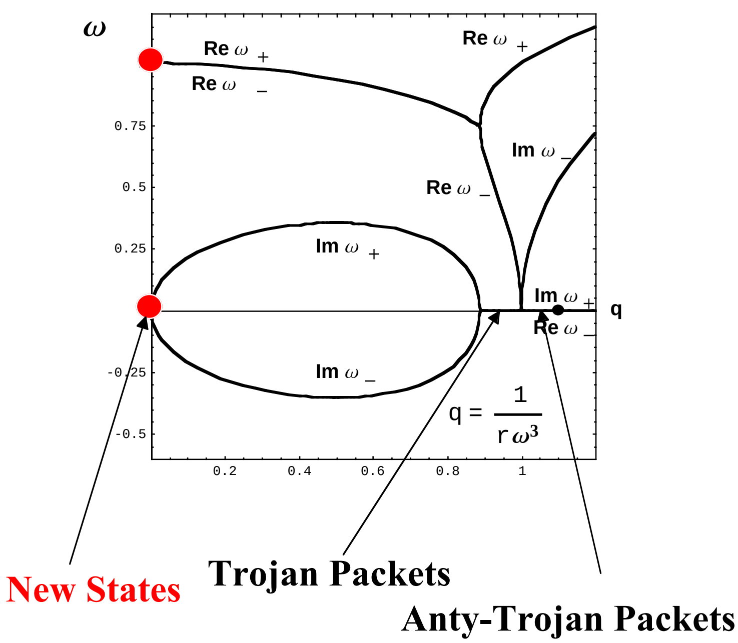

Fig. 1. Frequencies of small oscillations of the resonant circular orbit as the function of the parameter $q = 1/r^3\omega^2$. The orbit is stable for $8/9 \leq q \leq 1$. It formally regains stability for $q \to 0$ where the harmonic wavefunction is singular and we find the new states. The weakly unstable orbit supports non-dispersing wavepackets for $q < 1 + 3/4\omega^{-1/3}$.

been the inspiration for detailed investigation of new quantum resonances [5, 6] as well as for the discovery of highly correlated motions and their quantum counterparts in two electron atoms [7]. These Trojan wavepacets are the unique solutions of the time-dependent Schrödinger equation with the property that the probability density they describe represents the localized particle-like electron moving on well defined circular orbit. The generation of well localized Rydberg electron moving on a circular orbit is the goal of experimental Rydberg physics and the method of generation of those states provides the simplest and universal method. The detailed understanding of the Trojan dynamics in the time dependent field and frequency domain is the key to arbitary state generation of the hygrogen atom [8].

Unlike for any other atomic state the atom in the Trojan state contains well localized probability droplet which does not decay on the time-scale associated with the size of the electronic orbit. The driving CP electromagnetic field is not necessary to overcome radiative [9] friction, however it is essential for optical mixing of originally non-localized eigenstates of the atom. The ionization rates are also negligible for the parameter region where the corresponding classical motion is stable.

The main feature of Trojan localization is that the external field, while considered strong, is still a fraction of the atomic Coulomb field and therefore the quantum dynamics can be well described in terms of non-perturbed Coulomb states of bare single-electron atom. Brief understanding of this phenomenon can be achieved within a simple harmonic approximation [4] but more detailed studies show that this is a result of Chirikov universality which exhibits itself quantum mechanically [10].

In the following we describe new highly localized wavepackets moving around circular orbits, but when Coulomb manifolds can no longer be used to span the field-dressed wave

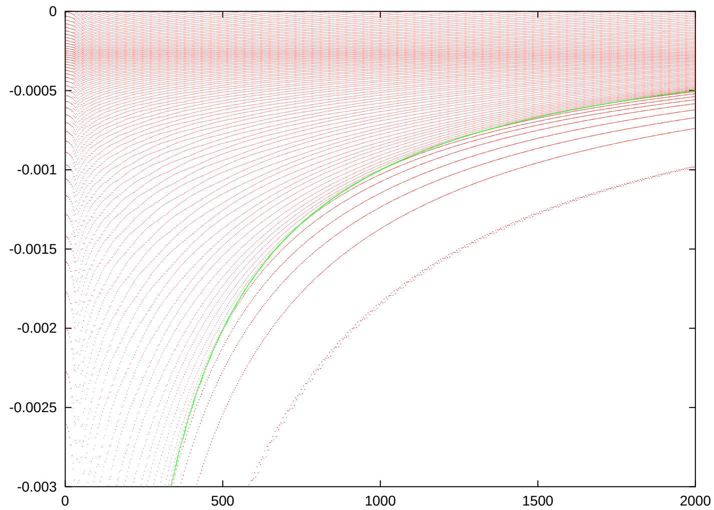

Fig. 2. Eigenvalues of the equation $[-d^2x/2dx^2 + V_{eff}(x)]\phi = E\phi$ for as functions of the potential radius a. States with the energy $E \approx V_{eff}(0)$ are strongly localized around $x = 0$ (the bold green line). The density of levels exhibists abrupt change across the energy line $E = V_{eff}(0)$

functions. They are the consequence of coherent rescattering of partial waves from the circularly moving Coulomb potential.

The harmonic stability analysis of resonant circular orbits [4] leads to the following hamiltonian

$$H_{osc} = \omega_+ a^+ a - \omega_- b^+ b, \tag{1}$$

where a, and b are annihilation operators for collective excitations in the phase space, ω_-/ω, ω_+/ω are functions of the parameter $q = 1/\omega^2 r^3$ and r is the radius of the orbit. The circular orbit is stable (Fig. 1) for moderate field strength $\mathcal{E} < 0.11\omega^{4/3}$. Weakly unstable orbits are also capable to support non-dispersing wave packets for the fields $\mathcal{E} < 3/4\omega^{5/3}$. They have been called anti-Trojan as moving in anti-phase with Trojan states around the circular orbit. Note that the orbit regains formally the stability for ultra-strong field when $q \to 0$ which corresponds to very large circular orbit comparing to the size of field-free Coulomb orbit with the motion of the same frequency. We will look for long-living states in this limit.

We start our analysis from the time dependent Schrödinger equation in the Kramers-Henneneberger (K-H) frame [11, 12]

$$\begin{aligned} H &= -\frac{\bigtriangledown^2}{2} - \frac{1}{|\mathbf{r}|} + \mathbf{E}(t)\cdot\mathbf{r}, \\ H_{KH} &= e^{-i\alpha(t)\cdot\bigtriangledown} e^{id\alpha(t)/dt\cdot\mathbf{r}} H e^{i\alpha(t)\cdot\bigtriangledown} e^{-id\alpha(t)/dt\cdot\mathbf{r}}, \end{aligned}$$

$$H_{KH}\Psi = i\frac{d\Psi}{dt}, \qquad H_{KH} = -\frac{\bigtriangledown^2}{2} - \frac{1}{|\mathbf{r}+\alpha(t)|}, \qquad \frac{d^2\alpha(t)}{dt^2} = \mathbf{E}(t), \tag{2}$$

where $\mathbf{E}(t) = \mathcal{E}(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$ and $\mathcal{E}$ is the strength of the CP field. It describes the electron motion in a uniformly rotating Coulomb potential, with the singularity moving around a circle with a radius a. The Fourier decomposition of the K-H potential

$$\frac{1}{|\mathbf{r}+\alpha(t)|} = -\sum_n V_n(\mathbf{r},\omega) e^{in\omega t} \tag{3}$$

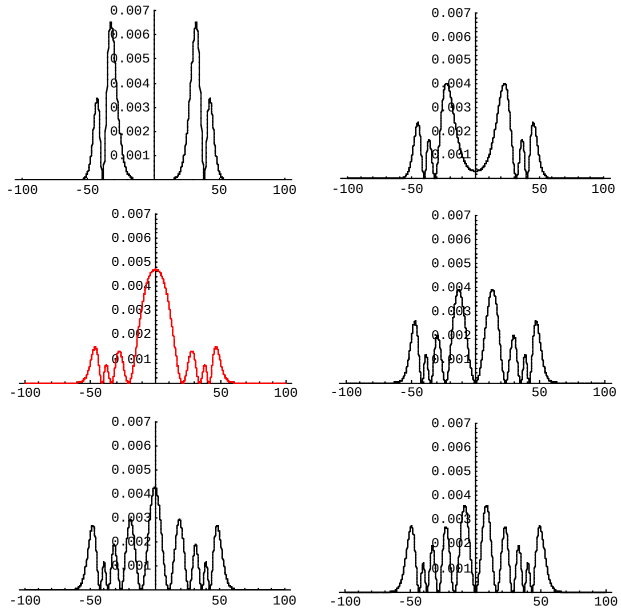

Fig. 3. First few eigenstates of the equation $[-d^2x/2dx^2 + V_{eff}(x)]\phi = E\phi$ for $a = 50$ a.u. The states for which the energy $E \approx V_{eff}(0)$ are strongly localized around $x = 0$ (the red plot).

leads to the stationary Schrödinger equation with the static potential [13]

$$\begin{aligned} V_{eff}(r) = V_0(r,\omega) &= -\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{|\mathbf{r}+\alpha(\tau)|}d\tau \\ &= -\frac{2}{\pi|r-a|}E\Big[-\frac{4ar}{(r-a)^2}\Big], \qquad a = |\alpha(t)|, \quad \tau = \omega t \end{aligned} \tag{4}$$

where E is the complete elliptic integral of the first type and $a = \mathcal{E}/\omega^2$. The latter expression allows to recognise the zero frequency component of K-H potential as the potential of a uniformly charged ring.

We look for the states localized around the center of the ring, at $r = 0$. They correspond to localized wave packets moving around a circular orbit in the laboratory frame. There is a strong intuitive argument for such localization. The zero-order component of the K-H potential extended in one dimension may be treated as a potential of a deformed quantum pendulum with no closed boundary conditions and the effective angle $\phi = \pi r/a$. It is able to support nonclassical states localized on the unstable equilibrium point with the energy close to the upside-down position or $E \approx V_{eff}(0)$ [10]. Those can be directly obtained from the WKB approximation being formaly singular around the energy of the separatrix of motions [10]. Fig. 2 shows the "spaghetti" of energy levels as the function of the radius a. States strongly localized are those with energies near the deformed separatrix $E = V_{eff}(0)$. Fig. 3 shows the localized eigenstate and some other eigenstates of the extended potential.

The quantum dynamics is basically harmonic around the $z = 0$ plane of motion and therefore we consider a two dimensional model of localization. While compared to the one dimensional case the localization is strongly enhanced due to the cylindrical symmetry. We start from the stationary Schrödinger equation

$$\Big(-\frac{1}{2}\frac{\partial^2}{\partial r^2} - \frac{1}{2r}\frac{\partial}{\partial r} - \frac{1}{2}\frac{m^2}{r^2} + V_{eff}(r)\Big)\phi = E\phi \tag{5}$$

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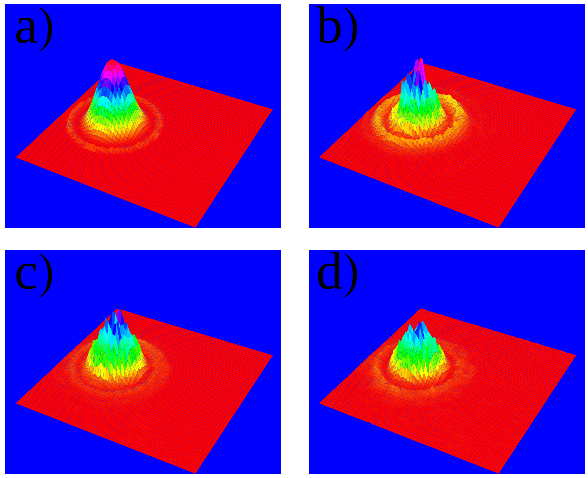

Fig. 4. Snapshots of the time evolution of the packet for $\mathcal{E} = 0.27$ and $\omega = 0.082$. a) $t = 0$, b) $t = 4$, c) $t = 9$ and d) $t = 19$ for the hydrogen in CP field. The radius of packet motion is $a = 40$ a.u. The movie (7.5MB) linked to this figure shows the full 20-cycle evolution.

We are interested in solutions with the maximum localization with $m = 0$. We also expect the solution to be the superposition of Coulomb continuum states and we postulate the approximation for the Coulomb potential of the ring as [14, 15]

$$V_{eff}(r) \approx Z\delta(r-a), \quad Z = -1/2\pi a \tag{6}$$

Equation (5) has the class of interesting solutions

$$\phi_k(r) = J_0(kr), \qquad J_0(k_n a) = 0, \qquad E = \frac{k_n^2}{2}, \tag{7}$$

where J_0 is the Bessel function of zero order. This is a discrete set of the contintinum states which are be being scattered by the ring potential with no phase shifts. The total scattering wave function for the delta ring can be expanded as

$$\phi_k(r) = \sum_m (-1)^m e^{i\delta_m} R_m(r) e^{im\phi}, \tag{8}$$

where

$$\begin{aligned} R_m(r) &= c_m J_m(kr) \quad r < a, \\ R_m(r) &= \gamma(k)(\cos\delta_m J_m(kr) - \sin\delta_m N_m(kr)) \quad r > 0, \end{aligned} \tag{9}$$

and N_m in the cylindrical Neumann function of order m. For (7) $\sin\delta_m = 0$ and this is the Ramsauer-Townsend effect [16] for this potential when the electron motion in the K-H frame can be considered as rescattering from the rotating target [17]. Strong localization occurs at $r = 0$ and the solution represents a well localized wave packet moving round the circular orbit in the laboratory frame from the construction. We are interested in the lowest continuum state of this kind namely the state with the smallest wave vector k. We expect it to be the best approximation for the states near the separatrix of true V_{eff} except for the cut-off normalization envelope

$$\Psi_k(\mathbf{r}t) = Ce^{id\alpha(t)/dt\cdot\mathbf{r}} J_0[k_0 r - \alpha(t)], \tag{10}$$

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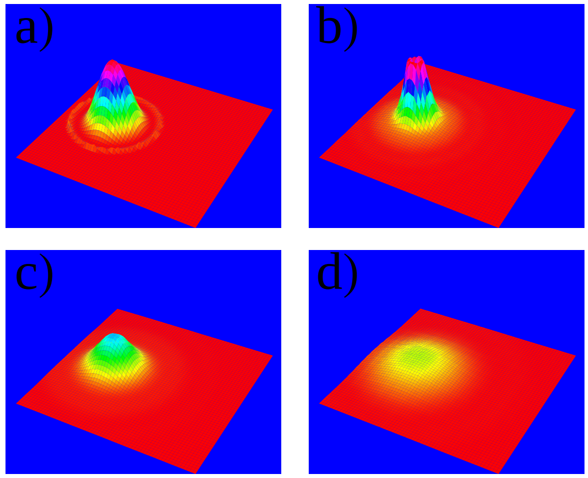

Fig. 5. Snapshots of the time evolution of the packet for $\mathcal{E} = 0.27$ and $\omega = 0.082$. a) $t = 0$, b) $t = 4$, c) $t = 6$ and d) $t = 9$ for the hydrogen in CP field and with removed Coulomb potential. The radius of packet motion is $a = 40$ a.u. The movie (7.5MB) linked to this figure shows the full 20-cycle evolution.

where $\alpha(t) = -\mathcal{E}/\omega^2(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$. In order to check our predictions numerically we have solved the exact time-dependent Schrödinger equation in two spatial dimensions using the split-operator method. We took the wave function (10) for $t = 0$ as the initial condition and the radial part was modulated by the slowly varying envelope to preserve the norm of the state. Fig. 4 shows the probability density during 20-cycle long time evolution. We observe clear trapping and focusing of the electron with small decay due to ionization. This is caused by higher-order terms in the expansion (3) which is negligible on the scale of the packet evolution and will be discussed elsewhere. The movie linked to Fig. 4 shows the whole 20-cycle evolution.

In order to demonstrate the co-operative action of the strong laser field and the Coulomb potential we have solved the Schrödinger for the same steady field envelope and the initial condition but with no atomic core. The packet then is a constant superposition of Volkov states for the CP field. Fig. 5 shows the corresponding probability density at certain time shots. Clear spreading is observed comparing to the case when the atomic core is present. The movie linked to Fig. 5 shows the 20-cycle time evolution. Note that while the leading energy shift of the Volkov superposition is equal to $-1/\alpha_0$ [18], namely the value of the Coulomb potential at the center of the packet, the long range global influence of the Coulomb tail in necessary to suppress spreading over the whole space.

In conclusion we have proved the existence of sharply localized quantum states much above the Chirikov stability threshold. They result from the inertia of quantum probability fluid coherently trapped by the rotating Coulomb potential. Unlike Trojan wavepackets they are trapped in continuum and cannot be easily generated from hydrogenic eigenstates. The simplest method of generation of those states beyond the generic alghorithm [19] whould be the excitation through resonant electron scattering and will be discussed elsewhere.

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